The National Curriculum and beyond

A Parents' guide to helping their children
to make the most of the learning opportunities
offered in the Primary School

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Chapter 14 Maths

Why is Maths given such a high priority in the primary school?

Put simply, maths is useful. We all use it countless times each day, in every area of our lives. On a personal level, we plan our time, we shop, cook, travel, look after the family finances and medical well-being, and so on; and when we are at work, maths is needed not only in professions such as accountancy, architecture, engineering, science and medicine, but also in retail, hospitality, administration, joinery, manufacturing, warehousing and so on. We might say that maths helps us to make sense of the world and describe it with precision, it enhances our ability to express ourselves clearly and unambiguously, and it contributes to solving problems. And in addition to these day-to-day uses, maths also helps the children to explore the world using a combination of logic and imagination, and to develop useful thinking and working skills such as self-organisation and care in their approach to tasks. It is for these reasons that Maths takes its place alongside English as one of the two most important subjects in the primary school.

For convenience, school maths is packaged under the headings of Number (which includes Calculations), Measurement, Geometry and Statistics, although these overlap at every turn.

Using maths to understand and describe the world: Maths offers the children, and ourselves, a whole range of concepts to help make sense of the world. It introduces the idea of numbers to describe precise quantities. It brings in the notion of size, and refines this so that the children understand concepts such as length, height, weight, time, volume, speed, temperature and so on, and these are combined with numbers to achieve accuracy. Calculations are used to describe actions and to predict the outcome of planned actions (how many sweets will we each get if we share this packet?). Common shapes are defined and given names. Positions are named. Movement is described in terms of direction and distance. And graphs allow us to consider large quantities of data in a pictorial format. Such ideas not only provide the children with the tools to understand and describe the world in the here and now, they also assist in many other areas of learning.

Using maths to aid communication: as the children learn more about the world, they naturally want to share their discoveries with friends and family. They normally use their everyday language for this (*I picked thirteen flowers for Mum this morning, six orange ones and seven yellow ones - they look really* *pretty together*).

However, maths also has its own very particular language, which allows for clear, concise, unambiguous, and even international, communication. This language can be spoken, of course *(six plus seven equals thirteen*), but also has a written form which makes use of numerals, signs and symbols (*6 + 7 = 13*), or could equally be shown as a diagram or a graph. Many mathematical words and symbols have passed into everyday use (for example, the numbers and the names of shapes), but others are used in very precise ways that may not match their everyday counterparts (for example, difference, squared, mean). The children learn to use this mathematical language correctly in both its spoken and written formats.

Using maths to solve everyday problems: a central part of the children's maths programme involves using the various elements to help them solve problems that they encounter as they go about their daily lives. Younger children might begin with problems such as: do I have enough money to buy a new rabbit hutch? which is the quickest way to Nan's house? when should I put the potatoes on, for tea at six o'clock? Older children will solve much more complex problems which might draw on both graphs and measures, and require several calculations using numbers into the thousands.

Of course, maths can't answer every problem, but it can often make a useful contribution. The children build a repertoire of problem-solving strategies which can be applied to everyday questions and situations. If in doubt, they can step through a checklist to help them: is maths going to help me with this problem? if so, what information is needed? exactly what maths is likely to provide that information? what steps do I need to take, and in what order? would any mathematical equipment be useful? They then need to do the maths. And afterwards: has the answer solved my original problem? The children are also taught the useful strategy of dividing up a large problem into a series of smaller ones, which can be solved separately and combined later.

Exploring the world through logic and imagination: from its very earliest stages, maths as a subject is both logical and consistent, making it an ideal environment in which to develop and practise reasoning skills.

As well as using reasoning to unravel problems and solve mathematical puzzles, the children also carry out so-called investigations. These might be of a general nature, such as, what is the relationship between children's height and weight? At other times, investigations are used to explore particular aspects of maths itself, for example: how could a packet of 10 chocolate buttons be shared between two children? Or, pick a number between 5 and 10, then double it, then halve it: what do you find? can this be done by halving first? does it work for odd numbers as well as even numbers? what about numbers greater than 10? As in real life, many of the questions will turn out to have several acceptable solutions. By exploring situations like these, the children learn to use their imagination and offer hypotheses, and then to test them out logically and honestly, using structures such as *if . . . then*.

Towards the end of their primary years, the children are taught that letters can sometimes be used to describe a general case; this is the beginning of algebra, of course, but it is not all that scary - at this level, the children might describe the number of matchsticks needed to build squares as *x lots of 4*, where *x* represents the number of squares we wish to build, or they might be asked to find all the numbers that make the number sentence $a + b = 7$ true.

Again, the children are encouraged to use their maths skills to explore other areas of the curriculum (for example, distances, areas and populations in geography), and to use them whenever they would be helpful in any area of their life.

Using maths to develop general thinking and working strategies: as they progress, the children become adept at answering questions by drawing on the facts they have learned and remembered. However, some problems are more complex, and may require strategies such as *estimating* results beforehand (so that wildly inaccurate responses can be dismissed), *simplifying* difficult tasks (perhaps by rounding numbers to make them easier to work with), inching towards a solution through *trial and improvement*, seeking out and *recognising patterns* in the data, and *making connections* between seemingly unrelated pieces of information. The children first need to be persuaded of the value of using such strategies, then they are given plenty of time, plenty of examples, and plenty of prompts to use them in their own work, both in maths and elsewhere.

Knowledge, skills and understanding

Maths is a hierarchical subject: every fact, every concept and every skill both builds on previous learning, and acts as a bridge to future learning.

The *knowledge* element in maths includes all the facts the children need to remember: basic things, such as the names of the numbers and how to write them; the names of signs and symbols; the names of measures, such as grams, kilometres, degrees (after all, we can't call them all *thingies*, can we?); the number of pennies that make a pound, and grams that make a kilogram; the number of sides on a hexagon; which is the x-axis on a graph, and so on. It is extremely useful to have speedy recall of these facts, and of others such as doubles, halves, multiplication tables and number bonds (e.g. 4 + 3 = 7, 3 + 4 = 7, 7 - 3 = 4, 7 - 4 = 3), as fluency when recalling facts allows the children to give their full attention to other aspects of the problem.

The children learn a wide range of mathematical *skills*, which are variously described as *routines, methods* and *procedures*, for example, how to set out *sums* on the page, how to carry out calculations of different sorts, and how to use mathematical equipment such as rulers, set squares and thermometers. Of course, there are also higher order skills such as reasoning skills and knowing how to tackle complex problems, as mentioned earlier. It is probably worth repeating here that mastery of these skills is not an end in itself; rather, it is a matter of helping the children to develop a repertoire that they can use to explore and make sense of the world and solve problems.

Taken together, these facts, rules and methods enable the children to work fluently and accurately on routine exercises. However, it is the children's *understanding* that helps them to decide just what maths they need to carry out in order to solve a problem. Each area of maths (number, calculations, measurement, geometry and statistics) presents the children with a particular set of concepts that need to be understood. For example, what numbers actually mean; what division is all about; what we are actually measuring when we talk about weight, area, volume and so on; what a *turn* is, and why it is so important in geometry; what sort of information is presented on different sorts of graphs.

Teachers often introduce new facts, skills and concepts by setting the children a problem which requires the new teaching-point to achieve a solution. For example, when the children are learning about *square metres* as a unit of *area* (the *teaching point*), the teacher might equip them with metre-square frames (quadrats) and ask them to investigate how well various species of wild plants grow in different parts of the school grounds (the *problem*). This practical activity will inevitably lead to a discussion of why the frames were used (i.e. to keep sample sizes consistent), why they were of this particular size, and where else we might find this measure useful. The teacher might link this problem to other areas of maths, by asking, for example, how many square metres would fit on the classroom floor (which might involve multiplication), and how they could present their findings on a graph.

Signs and symbols

The great virtue of maths is that it allows us to describe the world in a short-hand way, by extracting the essential maths from events and situations both in stories and in real life, and then using it again and again in similar circumstances. This is both economical and extremely useful. For example, when in the real world we see a family of eight rabbits, a mathematician would simply use the number *eight* (and the numeral ***8***) to represent this group. Clearly, there is no direct relationship between the concept of *eight* and the *rabbits* themselves. However, the same number and numeral can be used later to denote eight trees, children, elephants, mice, lines and so on, things of all different shapes and sizes, but all still denoted by the same number and numeral. Each of the signs and symbols used in maths can also be generalised in this way.

However, whilst this abstract nature of maths is undoubtedly one of its greatest assets, it also provides one of the greatest obstacles to the children, because they need to learn this specialised language and the symbols associated with it. There are numerals, also called digits, which unfortunately can mean different amounts depending on where they are found in any particular number (for example, the numeral ***2*** means a completely different quantity in the numbers 0.02, 0.2, 2, 23, 267, 2325, ½, $^2/_3$, 3^2). Then there are the symbols that represent actions: **+ - x ÷** (more on these later). And symbols such as the = sign, which means *is equal to*, and the < and > signs, which mean is numerically *less than* or *greater than* another number (the wider end is always next to the bigger number, and the closed end points to the smaller number). There are signs that look like punctuation marks: the *decimal point* looks like a full stop, but serves to change the value of the numerals around it; and the *comma* has no mathematical significance whatsoever, it is just a mark we make to help us read big numbers. And then, quite soon after we teach the children the sound values of letters to help them with their reading and spelling, we begin to use letters in maths, such as ***r*** to mean *radius* (which is fairly logical), and x (or indeed any other letter) to denote *a number we don't know*. Some children cotton on quite quickly; others find it much more challenging. However, this is the essence of maths; it is a code which the children need to crack before they can reap the full benefits of the subject.

In the following sections we come to the nitty-gritty of what the children learn in each area of maths. It is perhaps worth reiterating here that the facts, skills and understanding described here are merely building blocks, which need to be combined, linked and manipulated so that the children might use the whole of maths to their advantage.

Number

As the children are introduced to new numbers, they learn to read them (both as numerals and as words), to write them (again, in both formats) and to include them in their counting. Equally important, however, is that they fit them into their burgeoning mental picture of the number system, so that they understand how big (or small) it is, and where it lies in the sequence.

The children's understanding of numbers is constantly evolving: when they are just five years old, they are usually comfortable with whole numbers, and *ten* seems like a lot; by the time the children leave primary school, their personal number system will extend from one thousandth of a unit (0.001) through zero, to ten million (10,000,000) and beyond. It will eventually include:

~ *integers* (whole numbers), beginning with numbers from 0 - 9, and extending through tens, hundreds and thousands to millions

~ the concept of *place value* (the idea that digits take on a different value according to their place within a number), as in the example with the numeral 2 above

~ the idea that numbers can be presented in different ways e.g. 12 can be *partitioned* as 6 and 6, or 9 and 3, or 10 and 2. The children find this particularly useful when partitioning numbers into their separate hundreds, tens and ones, so 234 can become 200 and 30 and 4; or when carrying out a subtraction and recognise that 34 can be partitioned into 20 and 14

~ *fractions*: that is, *fractions of objects* (e.g. I ate half a pizza), noting that, unlike in real life, mathematical fractions must always be exactly equal shares; and *fractions of numbers* (half of the 180 children in school are boys, that is 90 boys). The children learn about simple fractions (e.g. $^1/_4$), improper fractions (e.g. $^5/_4$), mixed numbers (e.g. $1^1/_4$), and equivalent fractions (e.g. $^2/_5$ is the same as $^4/_{10}$)

~ *decimal fractions*: that is, using figures after a decimal point to indicate tenths, hundredths and thousandths of a unit; converting fractions to decimals and back again (e.g. $^{1}/_{5}$ is the same as 0.2; 0.37 is the same as $^{37}/_{100}$); knowing by heart the decimal equivalents of ½, ¼, ¾ and fifths; rounding recurring numbers (e.g. $^{1}/_{3}$ is the same as 0.333); using decimals in measurements that are based on 10s, 100s and 1,000s e.g £1.35, 1.375 litres

~ *percentages* (the number of parts per 100), and linking these to both fractions and decimals

~ numbers with particular properties: *odds, evens, squares* and *primes*, and finding and checking these, using multiplication tables

~ *negative numbers* (numbers below zero), as used to describe temperatures below freezing, or a poor *goal difference* in football league tables

~ *number sequences*: the idea that numbers can be displayed, in order, along a *number line*. These can be consecutive numbers such as 3 4 5 6 7; non-consecutive numbers such as 0.8, 3, 46, 127; counting forwards and backwards using jumps of any size (e.g. in twos or fives, or by doubling, or by adding a half at each step); and crossing zero when the children are ready

~ *ordinal* numbers (first, second, third etc) that describe an order, and the corresponding *cardinal* numbers (one, two, three, etc) that we use for counting

~ particular mathematical words and their precise meaning: *number, digit, numeral, numerator, denominator*

~ *factors* (pairs of numbers that multiply together to make a larger number: both 2 x 6 and 3 x 4 make 12, so 2, 3, 4 and 6 are all factors of 12), and *common factors* (factors that are shared by different numbers: the numbers 12 and 30 share the factors 2, 3 and 6)

~ *multiples* (numbers that appear in the multiplication table of a smaller number), and *common multiples* (numbers that appear in more than one table, and are therefore common to both)

~ the idea that we don't always have to be completely accurate: sometimes an *approximate* answer is sufficient (Q: How many people live in and around Chester? A: About 120,000); a judgement needs to be made as to how precise we need to be in any particular context

~ the idea of *rounding* numbers up or down to the nearest ten, hundred, thousand or ten thousand, to make calculations more manageable; any loss in accuracy is offset by the convenience

~ using letters and other symbols to represent missing or unknown numbers; for example, given the number sequence 2, 4, 6, ***x***, 10, the children would understand that the ***x*** represents the number ***8***

To return to the idea of *place value* for a moment: the children learn that our number system is based on columns which can hold no more than 9 items: any more and they spill over into the next column to the left. This is why the *value* of each digit is determined by its *place* in the number. In this system, any empty columns are filled by a *place holder* called *zero*. So the digit 2, by itself, denotes two units; if one hundred and ninety-nine more were added to it, we would have two hundred and one, which we write as 201: the zero is there just to hold a place, so that the 2 can take on the value of 200, rather than 20, which would be the case without it. In order to highlight the way our number system works, the children briefly examine the method used by the ancient Romans, who merely arranged the letters I V X L C D M in various ways to denote numbers, and had no use for columns or zeros; the children learn to recognise year numbers written in this way, for example MMXIV.

Numbers are probably the single most important ingredient in the maths curriculum, as they attach themselves to every other area: measurements, shapes, statistics and problem-solving. The upshot of this is that the children will need to remember everything they have been learning about numbers (in all their forms) and to use their knowledge and understanding judiciously when carrying out calculations of any kind.

Calculation

When used alone, numbers generally describe static situations, such as *there are nine fish in the aquarium*. However, life is much more dynamic than this, and when we want to plan for a future event, solve a problem or describe an action, we may need to reach for a calculation.

There are two parts to using calculations effectively. First, the children need to decide which calculation to use; this relies on understanding what each type of calculation really means, what sort of question it can answer, what it can tell them, and when they would use it. And second, they need to carry out the calculation accurately. Without both parts in place, any calculation they make just isn't very useful.

The children are introduced to four types of calculation: *addition, subtraction*, *multiplication* and *division*.

At its simplest level, *addition* describes situations where groups of objects or measurements are brought together, either in reality or hypothetically. The calculation helps us to find out how many objects there are altogether, or what the total measurement is. We could just count them of course, but addition is quicker. We use words and phrases such as *add, altogether, put together, makes, in total*.

Subtraction is a little more tricky, because it can be used in two distinct ways. In its *take away* form, objects are removed from a group, and the calculation is used to help us find out how many are left; the children use words such as *take away, minus, subtract, left*. The other form is called *comparison* or *difference*, and here the children use subtraction to compare the size of two groups of objects or two measurements, and find out how many more, or how many less (or *fewer*) there are in the one than in the other, how much *bigger / smaller / heavier / lighter* and so on.

The children are taught that *multiplication* is essentially a quick way of doing addition when the groups of objects or the quantities that are brought together are all the same size. As in addition, the aim is to find the total number of objects or the total measurement. So instead of writing 2 + 2 + 2 + 2 + 2, we can write 5 x 2, which is usually read either as *five times two* or *five twos*. Initially this is explained as meaning *five sets of two all added up together* or *five lots of two all added up together*; in some schools the number sentence is written as 2 x 5 and read as *two, five times*. Either way there are still ten objects. When used with fractions, the multiplication sign is read as of (½ x 6 = 3).

Division has two types of story. In the first, called *grouping*, the children consider how many smaller groups of a particular size can be made from one larger group of objects; for example, our hens have laid 30 eggs, how many egg-boxes will we need? (that is, *how many sixes can I make out of 30*? or *how many sixes are there in 30*?). In the second kind of story, called *sharing*, the children share a group of objects evenly among a number of recipients, and use the calculation to determine how many they will each receive; for example, *when six children share a bunch of 30 grapes, how many grapes will they get each*? Remainders can cause confusion: the children need to return to the original question if they are to interpret the remainder in the right way.

Carrying out calculations can be a tricky business, and the children move through a number of early stages before doing any formal *sums* at all. If, for example, they were to explore a situation where three ducks from one family and two from another family go for a swim together on the local pond, they might initially illustrate the event by using toy ducks and a bowl of water, and counting the ducks to find out how many joined in this escapade; later they might draw a picture of a pond and use plastic counters to represent the ducks; with more experience they will sketch both the pond and the counters, and draw arrows to indicate the ducks' movement; with a lot of guidance they will eventually move on to represent the story with a number sentence: 3 ducks and 2 ducks is 5 ducks; and finally they will reduce this to the abstract mathematical sentence: 3 + 2 = 5. It is important that the children understand that the numbers represent objects (or measurements, depending on the story).

The children work through similar steps as they develop skills in subtraction, multiplication and division, always keeping an eye on the concept, so that the children really understand what the calculation is telling them.

Even when the children reach the stage of working sums out in written format, a range of methods is taught for each type of calculation, so that they can select the most suitable one for the problem they face.

Clearly, we use some calculations more than others, and it makes sense to learn by heart the ones we use most often: after all, 6 + 7 is always 13, and 6 x 7 is always 42, whatever the context. So the children learn addition and subtraction facts, first to a total of ten and then to twenty, and gradually learn multiplication and division facts up to 12 x 12. These are variously called *number bonds, number facts, addition and subtraction facts*, and *multiplication facts*. Whilst these fingertip facts may be enough by themselves in certain circumstances, they are also invaluable when combined with more complex written methods of calculation.

When adding, the children learn mental methods such as memorising number bonds; counting-on using fingers; adding whole tens, hundreds and thousands; adding tens and then ones to solve problems such as 67 + 25; adding a hundred and taking one off (to add 99). They then move on to pencil-and-paper methods, such as making a note of running totals in horizontal sums; writing numbers in columns and adding the ones and then the tens etc; using this same column method and carrying figures from one column to the next when a column overflows; adding decimals using columns (including measurements such as 2.875 kilograms); and adding fractions together, even those with different denominators, such as $^2/_3 + ^1/_6$. The children learn that numbers can be added in any order without affecting the result. In each case, they are encouraged to keep sight of the story that the calculation represents.

Although subtraction has two distinct stories, their methods of calculation are interchangeable. The children learn to subtract by counting back (the fall-back method for *take away* problems); counting on (the default for questions of *comparison*); taking whole tens and hundreds away; writing the numbers into columns and taking away, first without exchanging units and then doing so, as per the approved methods. [Please see Chapter 21.] They also subtract fractions, decimals, percentages and so on. The children learn that, unlike addition, numbers cannot be subtracted in any order without affecting the result (clearly, 3 - 5 is different from 5 - 3). Again, there is a clear emphasis on remembering what the calculation is really about, and making the story make sense.

In the early stages, the children deal with multiplication much as they would addition. Then they learn to count in 2s, 3s, 5s and 10s, and to build up instant recall of all multiplication facts as far as 12 x 12. They make use of the principle that, for example, *4 threes* gives the same result *as 3 fours*. They also explore multiplying by 1, 10, 100, 1,000, 10,000, and 0, and when this has been understood, they carry out short- and long-multiplication in columns using the approved written methods. Towards the end of the primary school, they also learn to multiply fractions, and learn about multiples, factors, factor pairs, square numbers and cubed numbers.

Strictly, division is a form of *repeated subtraction* (that is, subtraction of the same number over and over again), in the same way that multiplication might be described as *repeated addition*. Both division stories, grouping (the eggs) and sharing (the grapes), can be calculated by performing the same subtraction sum: 30 - 6 - 6 - 6 - 6 - 6 = 0. This tells us that we can box-up five groups of 6 eggs, with none remaining; and it also tells us that, because each of the six children can have one grape every time we take a group of six grapes, they can have five grapes each. As you can see, the sharing story is rather like the grouping story, but adds a distribution episode at the end. Alternatively, instead of repeatedly taking away, the same result can be achieved by building up to the number: *6, 12, 18, 24, 30, that's 5 sixes*. So we can also use multiplication to solve division problems. As in all calculations, the children are encouraged to use mental methods in the first instance, and to fall back on short- and long-division written methods only when necessary. A strong link is established between division and fractions, as the children calculate halves, thirds, quarters, fifths and tenths, of objects (pizzas always give a clear picture), and of numbers and quantities.

Multiplication and division topics throw up lots of interesting questions and ideas: about *fairness* (division gives only an arithmetic version of fairness); about *remainders*, which always need to be interpreted in the context of the question; about *ratios* (in my recipe for making punch, I use one measure of vodka to three measures of fruit juice, that's a ratio of 1:3, that is *one to three*); proportions (in the same story, we can say that the proportion of vodka in the punch is one part in every four, that is *one in four*), and *scaling up* and *scaling down* (e.g. doubling or halving recipe quantities).

Maths is loaded with conventions that the children need to learn if they are to use it to their advantage. An important one when carrying out calculations is that the elements of a sum written in horizontal format need to be dealt with in a particular order. Take, for example, the number sentence 3 + 5 x 2 = ? If we read it from left-to-right, the answer is 16 (3 + 5 = 8, and 8 x 2 = 16). However, a maths convention dictates that any multiplication elements need to be completed before any addition or subtraction is carried out: work out 5 x 2, that's 10, now add the 3 so the answer is actually 13. This convention is summed up by the acronym BODMAS: first work out any numbers in ***b***rackets, then numbers separated by ***o***f, then ***d***ivision, ***m***ultiplication, ***a***ddition and ***s***ubtraction, in that order.

Towards the end of their primary years, the children also learn to use calculators and spreadsheets to aid calculation. Great care needs to be taken, however, as procedures and conventions built into the programs need to be strictly adhered to if they are to be at all useful.

Measurement

A good deal of the maths we use in our everyday lives involves measurements: we use measures to *describe people and objects* (my sunflower is over two metres tall); to *organize ourselves* (Nan's arriving in an hour); to *compare things* (this clarinet costs more than that flute); to *solve problems* (have we got enough orange juice for the party?); and to *aid our creativity* (the cake will need 25 minutes at 180^{o}C to achieve just the right taste and texture). In order to become really useful, measures often need to be combined with numbers and calculations of one sort or another.

So the children learn about:

- ~ *distance*: including *length, width, breadth, depth, perimeter, circumference* and longer *distances*; they use millimetres (mm), centimetres (cm), metres (m), kilometres (km), inches, feet, yards and miles
- ~ *weight* (also called *mass* if we want to be scientifically correct and exclude the influence of gravity): the children use grams (g), kilograms (Kg), tonnes (t), pounds (lb), stones (st) and tons
- ~ *time*: there are two distinct aspects to time; the first answers the question *when*? and the other is concerned with *how long*? When considering *when*? the children refer to day / night / morning / afternoon, yesterday / today / tomorrow, noon / midnight, clock-time in both analogue and digital formats, before / after, am / pm and 24-hour time. When talking about *how long*?, the children use units such as seconds, minutes, hours, days, weeks, months, years, decades, centuries and millennia
- ~ *money*: the children work with pence and pounds (and later euros); they also learn about prices, bills, change, value for money and budgeting (this topic links closely with Citizenship)
- ~ *area*: focussing on both regular and irregular *surfaces*, they use units such as square millimetres (mm^2), square centimetres (cm^2), square metres (m^2), hectares (ha), square kilometres (Km^2), acres and square miles
- ~ *capacity*: the size of a container, in terms of how much stuff, usually a liquid, it will hold; units include millilitres (ml), centilitres (cl), litres (l), pints and gallons
- ~ *volume*: the amount of stuff in that container; the children tend to use millilitres (ml), centilitres (cl), litres (l), pints and gallons for liquids, and cubic millimetres (mm^3), cubic centimetres (cm^3) and cubic metres (m^3) for solid shapes and objects
- ~ *temperature*: the children use degrees Celsius (e.g. 15^oC), including negative numbers for temperatures below zero

Towards the end of the primary years, the children are also introduced to so-called *compound measures*, such as *miles per hour*.

As each new measurement is introduced, the children first need to understand the quality that is being considered. For example, before they can discuss *weight* with any authority, they need to appreciate just what *heaviness* is, before they can talk about *area* they need to understand what a *surface* is, and so on. These concepts are learned through experience of handling objects and discussing what things look like and feel like. The children also learn to use words related to each quality, and to develop a sense of scale, e.g. what is the difference between *hot* and *warm*? when does *warm* become *cool*, and *cool* become *cold*? (many measures are *continuous* in this way). These words can be difficult to pin down, particularly because they often relate to a particular context (a *long* pencil is nowhere near as long as a *long* journey, for instance), so the children need to experience each quality in lots of different contexts, so that they appreciate what is normal in each.

The next step is to compare groups of three, four or five objects, using a method called *direct comparison*: that is, laying objects side by side, or weighing them in their hand or on a set of pan-scales, or pouring water from one container to the next until it overflows, and describing them as *longer, shorter, heavier, lighter, biggest, smallest* and so on.

Where objects cannot be compared directly, perhaps because they can't be moved close enough to each other, the children use an *intermediary*, which they can select for themselves, as an *informal unit of measurement*. For example, they might compare the size (the capacity) of two sacks by filling each of them to the top with footballs and then counting to see which sack contains more balls, or they might stick plastic blocks together to find out whether their bookshelf at home is tall enough to accommodate a particular book they have found in the library.

Of course, this method breaks down if the footballs, or the blocks, are of different sizes. So the idea of a *standard measure* is born, and the children are introduced to kilograms, metres, degrees, square metres, minutes and so on: measures which are the same size every time we use them. They explore each of the standard measures to get a feel for them: what does a *litre* look like (it can change its appearance dramatically, depending on the shape of the container in which it is held)? what does a *metre* look like? what does a *kilogram* feel like? Of course, most qualities have more than one unit of measurement attached to them, for example, *distance* uses kilometres, metres, centimetres and millimetres, depending on the scale. The children learn the name of each of these units, and how they are written both in their full form and in their abbreviated form (*metre* and *m, degrees* and ^{o}). Numbers are combined with the units of measurement to give greater accuracy: 12 centimetres, 3 litres, half an hour.

Through experience, the children build a mental picture of various *benchmarks* against which they can estimate new items. In terms of weight, this might be 10 grams (a teaspoonful of sugar), 250 grams (a tub of butter), a kilogram (a bag of sugar), their own weight, and so on. Estimation based around these benchmarks is a really useful skill, and demonstrates real understanding.

In order to achieve greater accuracy, the children learn to use a range of measuring equipment, and to do so carefully: rulers, tapes, weighing scales of various types, analogue and digital clocks, stop-watches, measuring jugs, thermometers, protractors and so on.

An equally important skill when measuring is to appreciate how accurate the measurement needs to be in any particular context: do we need to know that it is 3.56pm, or is it enough to know that it's coming up to 4 o'clock? is it important that we know exactly how much milk we have, or is it enough to know that we can last until we next go shopping?

The children also learn to apply their calculation skills to measures: how much will these two items weigh together? how much colder was it last night than during the daytime? how much will three loaves of bread cost? how much time will we each have on the computer if we take turns? how many days are there in three weeks? Calculations might need to be set out on paper, as a *sum*: in this case, the children may need to operate with up to three places of decimals (one litre and 375 millilitres would be written as 1.375 litres). Towards the end of their time in primary school, the children begin to use formulae to calculate the perimeter (merely a special sort of *distance*) and area of some regular 2-D shapes, including squares, rectangles, triangles, parallelograms, and the volume of cubes and cuboids.

In addition to the standard measures used across Europe, the children are introduced to imperial measures such as miles, feet and inches, stones and pounds, and so on. Using a graph, they also learn to convert measures between imperial and decimal units, for example miles and kilometres.

Geometry

Geometry divides itself into two main themes: *Shape* and *Position and direction*.

Handling plastic and wooden examples of both 2-dimensional and 3-dimensional shapes, the children undertake practical investigations to discover their essential qualities and properties: they name them, sketch them, identify them in drawings and in everyday objects, measure them, make patterns with them, build with them, construct them from specialist kits, and build them from nets. The children are helped to discover similarities and differences between them, and to identify the essence of each shape - what makes a hexagon a hexagon? What makes a cuboid a cuboid? Is a rectangle also a parallelogram? What exactly is a pyramid? What is the relationship between a square and a cube? And between a prism and a cylinder?

Using this approach, the children explore a whole range of 2-D shapes (*polygons*) in more detail, including circles and semi-circles, ovals, triangles (including right-angled, equilateral, isosceles and scalene variants), squares, rectangles, rhombuses, parallelograms, kites and trapeziums, pentagons, hexagons, heptagons and octagons. They find out how many sides each shape has; how long these are; whether any two or more of them are the same length, and if so, which ones; whether they are curved or straight; whether any two sides are parallel or perpendicular to each other; how many vertices (corners) it has, and the size of the angle at each (acute, right angle, obtuse or reflex, how many degrees); whether any two or more of these vertices are the same size, and if so, which ones; whether the shape is regular or irregular; and whether it has the property of reflective or rotational symmetry. The children make a special study of circles, learning the words *radius*, *diameter* and *circumference* and the mathematical relationships between them.

They learn that 3-dimensional shapes are called *polyhedrons*. These include cubes and cuboids, spheres and hemispheres, prisms and cylinders, pyramids and cones, as well as tetrahedrons, octahedrons, dodecahedrons and icosahedrons. When investigating these shapes, the children pose the same questions as they do in relation to 2-dimensional shapes (except that they substitute the word *edge* for *side*), and add further questions about their faces: how many are there? what 2-D shape are they? are any two or more the same shape? are any two or more the same size? and so on.

In every case, the children are encouraged to note where the shapes can be seen around them, both in nature and in manufactured objects, and to discuss whether the way they are used is in any way linked to their qualities and properties. For example, apart from aesthetic considerations, are there any advantages in having tiles that are square, rectangular, circular, oval or hexagonal? Why are most balls spherical, and what happens when they are not? Why are most walls perpendicular to the floor?

And an important new measure is introduced: the *angle*. The children are taught that an angle describes how far something or somebody turns round, using a scale rising to 360o for a full turn. The idea of an angle existing at the juncture of two lines / sides / edges (for example, at one of the vertices in a triangle) makes more sense to the children if they assume that the two lines used to lie on top of each other, and one remained still while the other turned to its new position. The children come to regard a quarter turn, that is, 90o or a right angle, as a benchmark measure that they see all around them in their everyday lives, and compare other angles with this, describing them as *less than a right angle* (an *acute* angle) or *more than a right angle* (*obtuse*). They also learn that the internal angles of all triangles add up to 180o, and those of a quadrilateral add up to 360o, and use this knowledge to deduce the size of unknown angles.

During the course of the primary school, the children learn to draw these mathematical shapes with increasing accuracy, using straight edges, rulers, set squares, protractors, curves and compasses, on both plain and squared paper. This requires them to make full use of their knowledge of the shapes' properties as they draw them.

In the other part of geometry, the children learn to describe *positions*, that is, where things are in relation to one another, and *directions* i.e. how things move from one place to another, either by turning, sliding or reflecting.

Position begins as a very unmathsy topic, as the children already use words and phrases such as *on, under, next to* and *in between* regularly in their everyday lives both in school and at home. They are encouraged to include measurements in order to gain greater accuracy (*the edge of the pond is 3 metres to the east of the wall*).

Later, the children use coordinates to describe a position on a grid, initially using letters and numbers (for example, *the treasure is in square D3*), and progressing to use only numbers that refer to the X - and Y-axes (*the treasure is in square 4,3*). They develop this idea further by drawing some of the 2-dimensional shapes they have been learning about onto squared paper, again using coordinates that refer to the X- and Y-axes; so a square might be made by joining points (1,2), (4,2), (4,5) and (1,5). Some children will progress further to include negative coordinates, for instance drawing a square using three given coordinates (1,-5), (4,-5), (4,-2) and understanding why (1,-2) needs to be the fourth point, in order to complete the shape.

But of course, things do not always stay in one position: they move, and in a particular *direction*. So the children build a vocabulary to describe the direction of a movement, such as *towards, over, into, through, forwards, backwards, left, right, clockwise, vertically, north, south-east* and so on. Again, these words and phrases don't appear particularly mathsy, but they are essential in everyday life, so it is as well to make sure that they are learned.

Moving on to investigate further: using 2-dimensional shapes, the children learn that objects move in one of three distinct ways: they can *reflect* (*flip over*), they can *rotate* (*turn on the spot*) or they can *translate* (*slide*).

There are four options when shapes *reflect*: they can flip vertically either upwards or downwards, or horizontally to the left or right. There are many more options when shapes *rotate*: they can turn about an imaginary pin either through their centre or through any one of their vertices; and in each of these scenarios they can turn clockwise or anticlockwise, and through a whole turn, a half turn, a quarter turn, or a particular number of degrees. The third way a shape can move is simply to relocate from one place to another. The children learn that this is called a *translation*. Shapes are said to translate *a particular distance in a particular direction*.

The children themselves move in each of these different ways, before directing their friends around a specified route, and instructing robots to do so (either as toys or on screen). They progress to working on squared graph paper, imagining how a geometric shape would move in a particular way, and plotting its new position.

Statistics

Most of the maths children use in their everyday lives involves just one or two numbers or measurements, and so they are able to describe the world around them, communicate information to others and solve problems mathematically in a fairly straightforward way using the concepts and calculations described in the earlier part of this chapter.

As adults, however, we are increasingly presented with data in the form of lists, tables, charts and graphs: for example, the results of surveys, or our monthly home electricity usage. These attempt to simplify large amounts of *raw data*, and convert them into *information* for us, so that we might recognise patterns and reach conclusions about the quantities they represent.

In preparation for this, the children are gradually introduced to a range of charts and graphs, progressing from pictures, tables and timetables, through tally charts, block graphs, bar charts, histograms, pictograms, line graphs, pie charts and scatter graphs, to Carroll diagrams and Venn diagrams. Some formats (including tally charts and block graphs) show so-called *discrete data*, that is, absolute numbers of people or objects; others (for example, a line graph showing how tall Jon was on each of his birthdays) show *continuous data*, where particular points on the graph are given, and the reader needs to deduce information about the periods in between.

In each case, the children learn to interpret the diagrams, first reading the titles and then taking note of any lines, blocks, shapes, colours, sizes and scales, variously to count, compare and aggregate quantities. They learn what sort of information can be shown on each type of chart: numbers or measurements, larger or smaller quantities, totals or proportions, one variable or more than one, and so on, and how reliable these might be.

At various times, the children design and conduct surveys among their classmates (perhaps, *which is your favourite genre in books*?), and then present their data in one of these graphical formats. They work their way through a process of sorting their data into categories, deciding on an appropriate graphical format, and then measuring, drawing, colouring, scaling and labelling their graph. Scaling can be particularly troublesome, so the children are encouraged to use intervals of 2, 5, 10 or 100 wherever this can be accommodated. Initially, the children create their charts and graphs by hand, to show that they understand the whole process, but because quantities can soon become unwieldy and because the process of drawing a graph accurately is extremely time-consuming, they swiftly move on feed their data into software which can display it in the graphical format of their choice. They are encouraged to review the charts and graphs produced in this way, to make sure they give the correct information.

The children learn that a second method of simplifying large sets of data is to reduce them to a single number. For example, in order to answer the question: a*re the children in Class 4 taller than those on Class 3*? they might measure each of the sixty children and calculate and compare the *mean* (average) height for each class. They need to make a judgement about how representative these single-figure simplifications are, and therefore how useful they might be.

What can we parents do to help?

Many children (and adults, too) are held back in their maths not because of shortcomings in their mathematical knowledge, skills or understanding, but by a lack of confidence in their ability to *do maths*. If we can persuade our children that maths is not separate from everything else in our lives, that it is merely a short-hand way of describing the world around them and the things they do, that it is a tool for helping them to get the most out of life, then they might avoid being intimidated by it. Remember, the jargon used in primary maths can always be restated using ordinary everyday words, phrases and sentences.

As parents you can:

~ make your children aware of the maths around them: numbers, measures, shapes, fractions, angles, graphs and so on

~ give home-based examples of the maths they are learning in school, for example, when they are learning numbers to 20, we can point out numbers on houses, busses, footballers' shirts, TV channels, prices, pages, clothes sizes, ages and so on; and we can count with them, counting both numbers alone and objects of various sizes (to show that 15 elephants might be bigger than 15 mice, but there are still 15 of each)

~ point out how you use maths yourself: when baking, cooking, doing DIY, filling the car with petrol, driving distances, giving medicine, buying paint, receiving change etc. Encourage them to help at all stages, both in the planning and in the doing

~ help them to understand the language of maths: whenever they meet technical words, and whenever they come across mathematical signs and symbols, translate them into ordinary English for them: remove the subject's air of secrecy (but do encourage them to use the formal words too, just so long as they really do understand them)

~ help them to remember and revise facts, such as number bonds, multiplication tables, the name of the units we use for each of the measures, the number of pennies in a pound and millimetres in a metre, the properties of particular shapes, the number of degrees in a full turn, whether we name coordinates as [X,Y] or [Y,X]

- ~ find out exactly how your school is teaching the different types of calculation; they are sure to have agreed methods, probably based on those set out in National Curriculum documentation [see Chapter 21]
- ~ help them with any homework they are given; encourage them to talk about what they are doing and explain it to you, so that you understand their method and are in a position to give advice if it doesn't work out as they thought it might; patience is essential
- ~ help them to manage their own pocket money
- ~ encourage whimsical questions and help them to investigate: how big would a punchbowl need to be if all my friends brought a bottle of juice and poured it in? How many hours of football do I play in a year? Encourage them to browse the facts and figures in one of the Books of Records, in order to satisfy their musings
- ~ play cards and board-games with them, which require mathematical understanding and skill
- ~ play logic and strategic games with them such as draughts and chess, to help them develop reasoning skills
- ~ check out computer-based resources - there are many excellent programs that children find extremely motivating

Maths is a logical subject, in that every new step builds on previous learning. However, we might need to look in different mathematical topics to find these building blocks. For example, children can only understand the relationship between litres and millilitres if they have already learned about numbers as large as thousands. When children *get stuck*, it is unusual for them to be completely and utterly in the dark; it is more likely that they have a *partial understanding* of how to reach a complete solution. You will need to talk around the question with your child to find out just where their problem lies:

- ~ is it that they don't understand the concept they need to use (or perhaps they didn't understand the step before this one, so that the new idea appears to be free-standing, rather than being based on something substantial)? If so, you need to explain the concept again in a context that is familiar to your child, so that it makes sense and, at the same time, they have a reliable example to which they can refer back when similar situations occur in the future. Many children benefit from pictures and diagrams to help them understand the concept.

~ have they perhaps not yet grasped the mathematical jargon, or the signs and symbols, that litter the pages of maths books? We need to help our children to link the symbols both to their mathematical name and to the everyday words that we normally use in general conversation: so the sign ***x*** could become *multiplied by,* or *times*, or *sets of*, or *lots of,* or *boxes o*f, or *bags of,* or *packets of*, and so on

~ maybe they have not yet appreciated the association between a calculation (or *sum*) and the type of situation in which it can be useful? If so, we need to reinterpret this sort of sum for them, and link the abstract with real-life events to which they can relate on a personal level; and then they will need lots of other examples of where this kind of sum can be helpful to them

~ have they just forgotten how to carry out a procedure (for example, should they work from left to right, or right to left)? Unfortunately, if the children have been taught a *routine* without understanding, and then forget what to do, they are well and truly stuck: in this case, they need first to understand the story involved in the calculation, and then work it through from first principles. Methods of calculation need to be both reliable and based on their understanding: for example, if they can't remember how to do a multiplication sum (247 x 3), then they should understand that they can always revert to the corresponding addition sum (247 + 247 + 247) and solve it this way

~ or are they just panicking and *freezing*? If this is the case, we need to tread very carefully as we explore the situation and find out just where the problem lies, and perhaps relay our findings back to their teacher